

**EXERCISES [MAI 1.2-1.3]**  
**EXPONENTS – SYSTEMS OF LINEAR EQUATIONS**  
**SOLUTIONS**

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**A. Paper 1 questions (SHORT)**

**EXPONENTS**

1.

$3^3 = 27$	$3^{-2} = \frac{1}{9}$	$3^{-3} = \frac{1}{27}$
$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$	$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$	$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$
$\left(\frac{1}{3}\right)^{-2} = 9$	$\left(\frac{2}{3}\right)^{-2} = \frac{9}{4}$	$\left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$
$4^{\frac{1}{2}} = 2$	$4^{\frac{3}{2}} = 8$	$4^{-\frac{1}{2}} = \frac{1}{2}$
$25^{\frac{1}{2}} = 5$	$25^{-\frac{1}{2}} = \frac{1}{5}$	$\left(\frac{25}{4}\right)^{\frac{1}{2}} = \frac{5}{2}$

2.

$3^2 \times 2^3 = 72$	$3^5 \times 3^{-3} = 9$	$3^{-5} \times 3^3 = \frac{1}{9}$
$3^0 + 0^3 = 1$	$\frac{5^4 \times 2^3}{2^2 \times 5^3} = 10$	$\frac{7^4 \times 7^2}{7 \times 7^7} = 1$
$3^{-1} \times 2^3 = \frac{8}{3}$	$3 \times 2^{-3} = \frac{3}{8}$	$\frac{2^{-1}}{3^{-1}} = \frac{3}{2}$

3.

$\frac{30^2}{15^2} = 4$	$\frac{15^{-2}}{30^{-2}} = 4$	$\frac{12^{\frac{1}{2}}}{3^{\frac{1}{2}}} = 2$
$7^{0.3} \times 7^{0.7} = 7$	$7^{\frac{1}{2}} \times 7^{\frac{3}{2}} = 49$	$7^{-2} \times 7 = \frac{1}{7}$
$7^2 \times 7^{-1} = 7$	$\frac{2^3 \times 3^3}{6^3} = 1$	$\frac{4^3 \times 3^3}{6^3} = 8$

4.

$a^6 a^2 = a^8$	$a^6 a = a^7$	$(a^{\frac{1}{2}})^6 = a^3$
$(a^6)^2 = a^{12}$	$(a^2)^6 = a^{12}$	$\frac{a^6}{a^2} = a^4$
$a^2 a^3 a = a^6$	$\frac{a^2 a^6}{a^5} = a^3$	$\frac{a^2 b^6}{b^2 a^{-3} b^4} = a^5$
$\left(a^{\frac{3}{5}}\right)^5 = a^3$	$\left(a^{\frac{3}{5}}\right)^{10} = a^6$	$(a^{-1})^{-2} = a^2$
$a^{x+3} a^{1-x} = a^4$	$\frac{a^{n+5}}{a^{n+3}} = a^2$	$\frac{a^{-8}}{a^{-10}} = a^2$

5.

$\frac{A^6 B^3 C^{10}}{C^5 A^2 B} = A^4 B^2 C^5$
$\frac{A^2 B + AB^3}{AB} = A + B^2$
$\frac{2A + A(4B) + (2A)^2}{2A} = 1 + 2B + 2A$
$\frac{A^4 B^3 + A^3 B^4}{A + B} = A^3 B^3$

6.

$e = 2.72$	$\pi = 3.14$
$2e = 5.44$	$2\pi = 6.28$
$e^2 = 7.39$	$\pi^2 = 9.87$
$\sqrt{e} = 1.65$	$\frac{\pi}{e} = 1.16$
$\sqrt[3]{e^2} = 1.95$	$\frac{e}{\pi} = 0.865$

7. 1 (d) 2 (e) 3 (c) 4 (b) 5 (g) 6 (a) 7 (f)

8. (a)  $2^{2x} = 2^{1-x} \Leftrightarrow 2x = 1 - x \Leftrightarrow 3x = 1 \Leftrightarrow x = 1/3$

(b)  $2^{2x} = 8^{1-x} \Leftrightarrow 2^{2x} = 2^{3-3x} \Leftrightarrow 2x = 3 - 3x \Leftrightarrow 5x = 3 \Leftrightarrow x = 3/5$

(c)  $4^{2x} = 8^{1-x} \Leftrightarrow 2^{4x} = 2^{3-3x} \Leftrightarrow 4x = 3 - 3x \Leftrightarrow 7x = 3 \Leftrightarrow x = 3/7$

(d)  $8^{x+3} = 16^x \Leftrightarrow 2^{3x+9} = 2^{4x} \Leftrightarrow 3x + 9 = 4x \Leftrightarrow x = 9$

(e)  $\frac{1}{2^x} = 4^{x-3} \Leftrightarrow 2^{-x} = 2^{2x-6} \Leftrightarrow -x = 2x - 6 \Leftrightarrow 6 = 3x \Leftrightarrow x = 2$

(f)  $\sqrt{2^x} = 4^{1-x} \Leftrightarrow 2^{\frac{x}{2}} = 2^{2-2x} \Leftrightarrow \frac{x}{2} = 2 - 2x \Leftrightarrow x = 4 - 4x \Leftrightarrow 5x = 4 \Leftrightarrow x = 4/5$

9. (a)  $25^{x+1} = 5^3 \Leftrightarrow 5^{2x+2} = 5^3 \Leftrightarrow 2x + 2 = 3 \Leftrightarrow 2x = 1 \Leftrightarrow x = 1/2$

(b)  $25^{x+1} = \frac{1}{5^x} \Leftrightarrow 5^{2x+2} = 5^{-x} \Leftrightarrow 2x + 2 = -x \Leftrightarrow 3x = -2 \Leftrightarrow x = -2/3$

(c)  $25^{x+1} = \sqrt{5^x} \Leftrightarrow 5^{2x+2} = 5^{\frac{x}{2}} \Leftrightarrow 2x + 2 = \frac{x}{2} \Leftrightarrow 4x + 4 = x \Leftrightarrow 3x = -4 \Leftrightarrow x = -4/3$

(d)  $7^{x^2-5x} = 1 \Leftrightarrow 7^{x^2-5x} = 7^0 \Leftrightarrow x^2 - 5x = 0 \Leftrightarrow x(x-5) = 0 \Leftrightarrow x = 0 \text{ or } x = 5$

10.  $9^{2x} = 27^{(1-x)} \Leftrightarrow 9^{2x} = 9^{3-3x} \Leftrightarrow 2x = 3 - 3x \Leftrightarrow 5x = 3 \Leftrightarrow x = 3/5$

11.  $9^{x-1} = \left(\frac{1}{3}\right)^{2x} \Rightarrow 3^{2x-2} = 3^{-2x} \Rightarrow 2x - 2 = -2x \Rightarrow x = \frac{1}{2}$

## SYSTEMS OF LINEAR EQUATIONS

12. (a)  $x = 4, y = 7$

(b)  $a = 3.7, b = 4.7$

13. (a)  $x = 4, y = 7, z = -2$

(b)  $a = 1, b = 2, c = 3$

14.  $x = 2, y = 5$

15.  $x = 2, y = 7$

16. (a)  $A+B+C=20$

$$2A+3B=41$$

$$A-C=7$$

(b)  $A=10, B=7, C=3$

(c) Indeed, the sum of 10, 7, 3 is 20

The sum of 2A and 3B is  $20+21=41$

10 is 7 more than 3